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Weight minimization of Speed Reducer design problem using PSO, SA, PS, GODLIKE, CUCKOO, FF, FP, ALO, GSA and MVO

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Abstract . The objective functions used in Engineering Optimization are complex in nature with many variables and constraints. Conventional optimization tools sometimes fail to give global optima point. Very popular methods like Genetic Algorithm, Pattern Search, Simulated Annealing, and Gradient Search are useful methods to find global optima related to engineering problems. This paper attempts to use new non-traditional optimization algorithms which are used to find the minimum weight of designing a speed reducer to obtain global optimum solutions. The weight, number of iterations and the total elapsed time to complete the problems are all compared using these ten non-traditional optimization methods.

Keywords: Pattern search , Simulate annealing, Pattern search, GODLIKE, Cuckoo search, Firefly algorithm, Flower pollination, Ant lion optimizer, Gravitational search algorithm, Multi-verse optimizer.

1. Introduction

The toothed are used as independent units to reduce or increase the speed and they are enclosed in rigid closed housings. The housings provide support for the shafts, hold lubricants inside, protect the gears from dust and moisture and give necessary cooling surface to dissipate the heat generated. When the unit is used as a speed reducing device, it is called speed reducer. Speed reducers are widely used for reduction of speed in turbine generator set, from motor to machine tools, in rolling mills from engine to road wheels in automobiles etc.

Nomenclature

- b - Face width of gear
- m - Teeth module
- z - Number of pining teeth
- l_1 - Length of shaft 1 between bearing
- l_2 - Length of shaft 2 between bearing
- d_1 - Diameter of shaft 1
- d_2 - Diameter of shaft 2

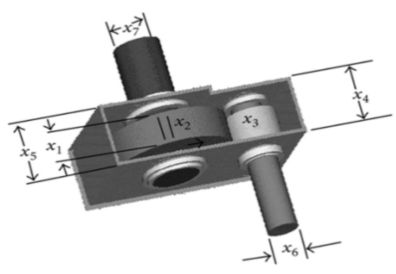

Fig.1Schematic of the speed reducer to be designed

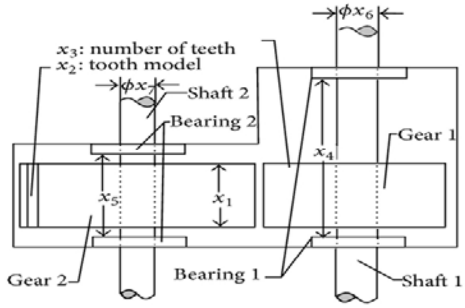

Fig.2 Speed reducer

2.1. FORMULATION OF PROBLEM:

The design of the speed reducer is considered with the face width (b) , module of teeth (m), number of teeth on pining (z), length of shaft 1 between bearings (l_1) , length of the shaft 2 between bearings (l_2), diameter of the shaft 1 (d_1), diameter of shaft 2 (d_2) respectively. The constraints include limitations on the bending stress of gear teeth, surface stress, transverse deflections of shaft 1 and 2 due to transmitted force, and stresses in shafts 1 and 2. The objective is to minimize the total weight of the speed reducer. The weight of the speed reducer includes both the weight of the gears as well as the weight of the shafts.

2.2.CONSTANTS: (Golinski's Speed Reducer)

C_1	0.7854	C_9	1.93	C_{17}	5
C_2	3.3333	C_{10}	745	C_{18}	12
C_3	14.9334	C_{11}	$16.9*10^6$	C_{19}	1.5
C_4	43.0934	C_{12}	0.1	C_{20}	1.9
C_5	1.508	C_{13}	1100	C_{21}	1.1
C_6	7.4777	C_{14}	$157.5*10^6$		
C_7	27	C_{15}	850		
C_8	397.5	C_{16}	40		

2.3.DESIGN VARIABLES (Leticia C.Cagnina and Susana C. Esquivel, 2008)

The seven design variables of the problem are as follows

b - Face width of gear (x_1)

m - Teeth module (x_2)

z - Number of pining teeth (x_3)

l_1 - Length of shaft 1 between bearing (x_4)

l_2 - Length of shaft 2 between bearing (x_5)

d_1 - Diameter of shaft 1 (x_6)

d_2 - Diameter of shaft 2 (x_7)

Optimization problem is defined as (Parashar.S., 2004) (AL-Oraby et.al.H., 2014)

Minimise $F(Gear\ \ box\ \ weight)$ Subject to

g_1 $(bending\ \ stress\ of\ \ gear\ \ tooth) \le 0.0$

g_2 $(contact\ \ stress\ of\ \ gear\ \ tooth) \le 0.0$

g_3, g_4 $(transverse\ \ deflection\ of\ \ shafts\ 1{,}2) \le 0.0$ g_5, g_6 $(stresses\ \ in\ \ shafts\ 1{,}2) \le 0.0$

$g_{7} - g_{9}$ (dimensional restrictions)

g_{10}, g_{11} (dimension requirements on the shafts)

2.4.CONSTRAINTS

Upper bound on the bending stress of the gear tooth
(Constraint 1)

$$\frac{C_7}{bm^2 z} \leq 1$$

$$\frac{27}{bm^2 z} \leq 1 \qquad (1)$$

Upper bound on the contact stress of the gear tooth
(Constraint 2)

$$\frac{C_8}{bm^2 z^2} \leq 1$$

$$\frac{397.5}{bm^2 z^2} \leq 1 \qquad (2)$$

Upper bounds on the transverse deflection of shaft 1
(Constraint 3)

$$\frac{C_9 \, l_1^3}{m \, z \, d_1^4} \leq 1$$

$$\frac{1.93 \, l_1^3}{m \, z \, d_1^4} \leq 1 \qquad (3)$$

Upper bounds on the transverse deflection of shaft 2
(Constraint 4)

$$\frac{C_9 \, l_2^3}{m \, z \, d_2^4} \leq 1$$

$$\frac{1.93 \, l_2^3}{m \, z \, d_2^4} \leq 1 \qquad (4)$$

Constraint on stress in the gear shaft 1
(Constraint 5)

$$\frac{\sqrt{\left(\frac{C_{10} \, l_1}{m \, z}\right)^2 + C_{11}}}{C_{12} \, d_1^3} \leq C_{13}$$

$$\frac{\sqrt{\left(\frac{745 \, l_1}{m \, z}\right)^2 + 16.9 \times 10^6}}{0.1 \, d_1^3} \leq 1100$$

(5)

Constraint on stress in the gear shaft 2
(Constraint 6)

$$\frac{\sqrt{\left(\frac{C_{10}\ l_1}{m\ z}\right)^2 + C_{14}}}{C_{12}\ d_1^3} \leq C_{15}$$

$$\frac{\sqrt{\left(\frac{745\ l_1}{m\ z}\right)^2 + 157.5\times10^6}}{0.1\ d_1^3} \leq 850 \quad (6)$$

Dimensional restrictions based on space and experience
(Constraints: 7 – 9)

$$\frac{m\ z}{C_{16}} \leq 1$$

$$\frac{m\ z}{40} \leq 1 \quad (7)$$

$$\frac{C_{17}\ m}{b} \leq 1$$

$$\frac{5\ m}{b} \leq 1 \quad (8)$$

$$\frac{b}{C_{18}\ m} \leq 1$$

$$\frac{b}{12\ m} \leq 1 \quad (9)$$

Design requirements on the shafts based on experience
(Constraints - 10, 11)

$$C_{19}\ d_1 + C_{20} \leq l_1$$

$$1.5\ d_1 + 1.9 \leq l_1 \quad (10)$$

$$C_{21}\ d_2 + C_{20} \leq l_2$$

$$1.1\ d_2 + 1.9 \leq l_2 \quad (11)$$

2.5. Variables bounds:

The upper bounds and lower bounds of design variables are

$$2.6 \leq b \leq 3.6$$

$$0.7 \leq m \leq 0.8$$

$$17 \leq z \leq 28$$

$$7.3 \leq l_1 \leq 8.3$$

$$7.3 \leq l_2 \leq 8.3$$

$$2.9 \leq d_1 \leq 3.9$$

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$$5.0 \le d_2 \le 5.5$$

	b - Face width of gear (x_1)		**m - Teeth module** (x_2)		**z - Number of pining teeth** (x_3)		l_1 **- Length of shaft 1 between bearings** (x_4)		l_2 **- Length of shaft 2 between bearings** (x_5)		d_1 **- Diameter of shaft 1** (x_6)		d_2 **- Diameter of shaft 2** (x_7)	
	Cm	mm	cm	mm	cm	mm	cm	mm	cm	mm	cm	mm	cm	mm
pper ound	3.6	36	0.8	8	28	280	8.3	83	8.3	83	3.9	39	5.5	55
wer ound	2.6	26	0.9	9	17	170	7.3	73	7.3	73	2.9	29	5	50
ptimum	3.500101	35	0.700008	7	17.00002	170	7.51772	75.1772	7.783269	77.8326	3.35084	33.5084	5.28679	52.8679

2.6.Mathematical formulation: (Afondo C.C.Lemonge,Helio I.C.barbosa Carlos C.H.Borges, Francilene B.S.Silva, 2010)

$$F(b,m,z,l_1,l_2,d_1,d_2) = C_1\, b\, m^2 \left(C_2 z^2 + C_3 z - C_4\right) - C_5\left(d_1^2 + d_2^2\right) + C_6\left(d_1^3 + d_2^3\right) + C_1\left(l_1 d_1^2 + l_2 d_2^2\right)$$

$$F(objective) = C_1\, x_1 x_2^2 \left(C_2 x_3^2 + C_3\, x_3 - C_4\right) - C_5\left(x_6^2 + x_7^2\right) + C_6\left(x_6^3 + x_7^3\right) + C_1\left(x_4 x_6^2 + x_5 x_7^2\right)$$

$$F(objective) = 0.7854\, x_1 x_2^2 \left(3.3333 x_3^2 + 14.9334 x_3 - 43.0934\right) - 1.508\left(x_6^2 + x_7^2\right) + 7.4777\left(x_6^3 + x_7^3\right) + 0.7854\left(x_4 x_6^2 + x_5 x_7^2\right)$$

Mathematical formulation: (Afondo C.C.Lemonge,Helio I.C.barbosa Carlos C.H.Borges, Francilene B.S.Silva, 2010)

The mathematical formulation of the objective function $f(X)$ is to minimize the weight of the speed reducer subject to the constraints of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shaft. The objective is to minimize the weight of the speed reducer design problem. The problem is:

Minimize :

$$f(X) = 0.7854\, x_1 x_2^2 \left(3.3333 x_3^2 + 14.9334 x_3 - 43.0934\right) - 1.508\left(x_6^2 + x_7^2\right) + 7.4777\left(x_6^3 + x_7^3\right) + 0.7854\left(x_4 x_6^2 + x_5 x_7^2\right)$$

subject to

$$\frac{27}{x_1 x_2^2 x_3} \le 1$$

$$\frac{397.5}{x_1 x_2^2 x_3^2} \le 1$$

$$\frac{1.93\, x_4^3}{x_2\, x_3\, x_6^4} \le 1$$

$$\frac{1.93\, x_5^3}{x_2 x_3 x_7^4} \le 1$$

$$\frac{1}{110\, x_6^3} \sqrt{\left(\frac{745.0\, x_4}{x_2\, x_3}\right)^2 + 16.9 \times 10^6} \le 1$$

$$\frac{1}{85\, x_7^3} \sqrt{\left(\frac{745.0\, x_5}{x_2\, x_3}\right)^2 + 157.5 \times 10^6} \le 1$$

$$\frac{x_2\, x_3}{40} \le 1$$

$$\frac{5\, x_2}{x_1} \le 1$$

$$\frac{x_1}{12\, x_2} \le 1$$

$$\frac{1.5\, x_6 + 1.9}{x_4} \le 1$$

$$\frac{1.1 x_7 + 1.9}{x_5} \le 1$$

Explicit bounds on design variable :

$$2.6 \le x_1 \le 3.6$$

$$0.7 \le x_2 \le 0.8$$

$$17 \le x_3 \le 28$$

$$7.3 \le x_4 \le 8.3$$

$$7.3 \le x_5 \le 8.3$$

$$2.9 \leq x_6 \leq 3.9$$

$$5.0 \leq x_7 \leq 5.5$$

3. Comparative Results

The ten methods are run 20 trails and the average is taken and the results were compared.

Face width of gear (x_1)

Teeth module (x_2)

Number of pining teeth (x_3)

Length of shaft 1 between bearings (x_4)

Length of shaft 2 between bearings (x_5)

Diameter of shaft 1 (x_6)

Diameter of shaft 2 (x_7)

Comparative Results

Trial No	PSO	SA	PS	GL	Cuckoo	FF	FP	ALO	GSA	MVO
X_1	3.500101	3.500291	3.5	3.508781	3.500017	3.50001	3.500004	3.5	3.559327	3.521317
X_2	0.700008	0.7	0.7	0.700511	0.7	0.7	0.7	0.7	0.706531	0.7
X_3	17.00002	17.38547	18.17172	17.01117	17	17	17	17	19.44552	17
X_4	7.51772	7.839092	7.3	7.712554	7.3	7.794979	7.300051	7.698099	7.904567	7.530287
X_5	7.783269	7.975486	7.715146	7.933927	7.715927	8.024645	7.715378	7.917097	8.024617	8.017789
X_6	3.350842	3.357579	3.349371	3.355449	3.350315	3.355642	3.350218	3.350991	3.566817	3.454274
X_7	5.286797	5.286776	5.286492	5.290331	5.286692	5.286774	5.286657	5.286724	5.350509	5.287023
Weight	**3145.922**	3223.153	3350.404	3162.799	3142.226	3154.947	3142.159	3150.358	3799.345	3191.186
Time	0.817205	30.84923	0.879797	10.39364	15.78775	8.008205	6.479024	62.66622	14.56511	6.759705
Iteration	200	20760	3	67106	100000	20000	2000	1000	1000	1000

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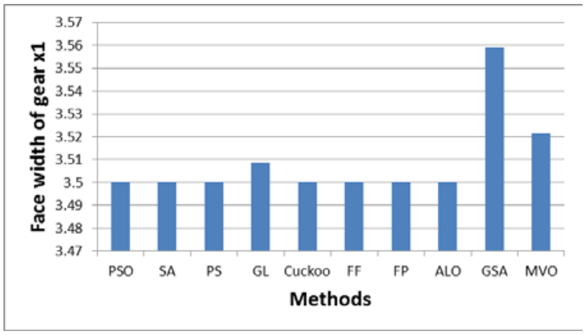

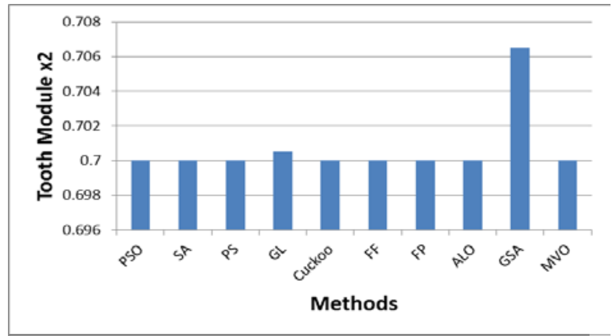

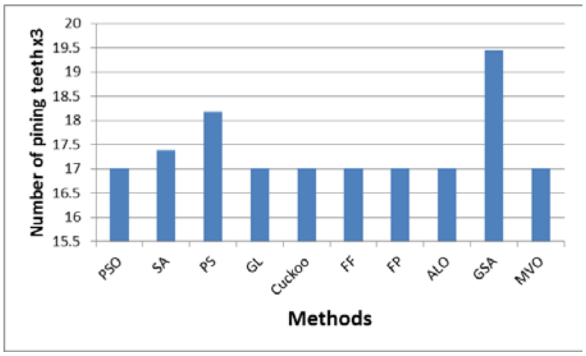

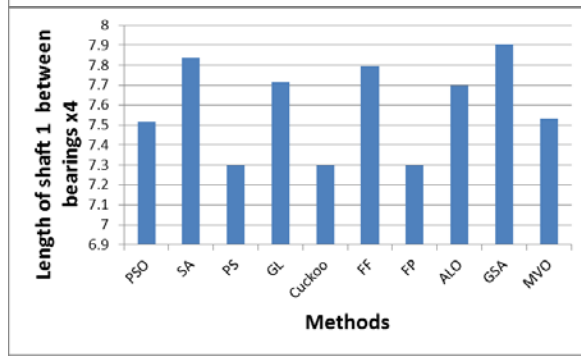

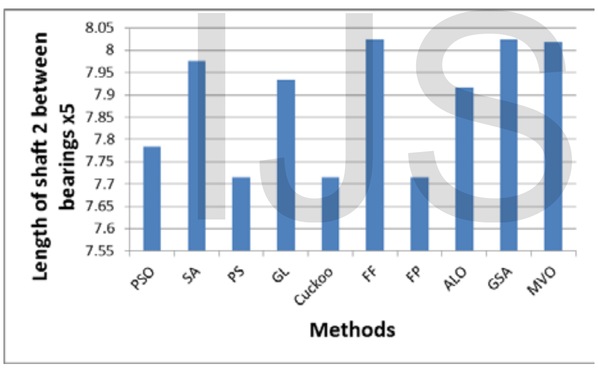

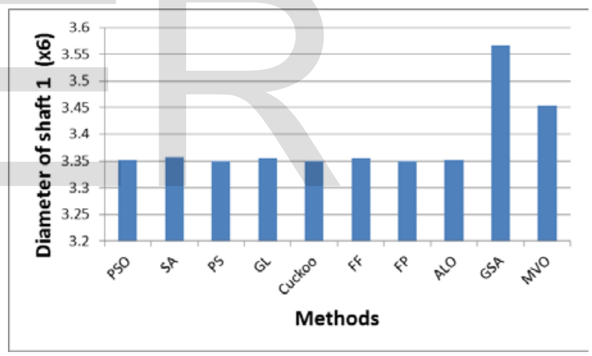

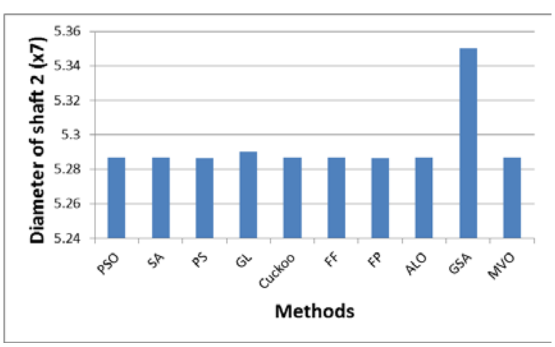

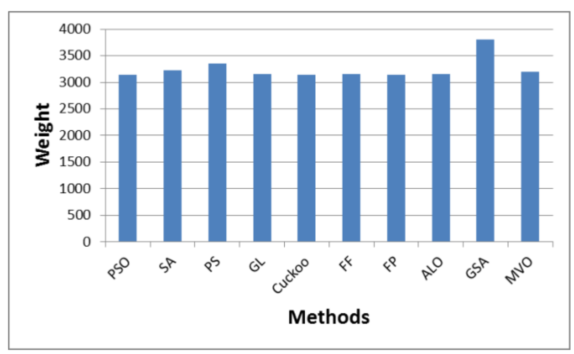

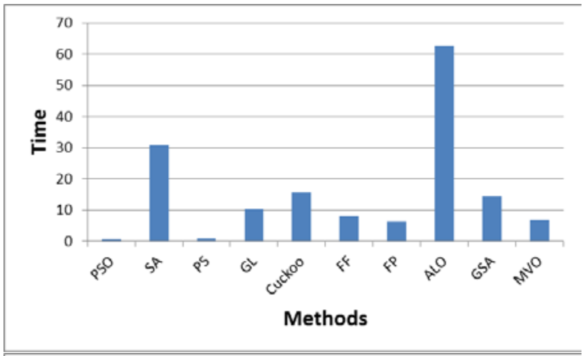

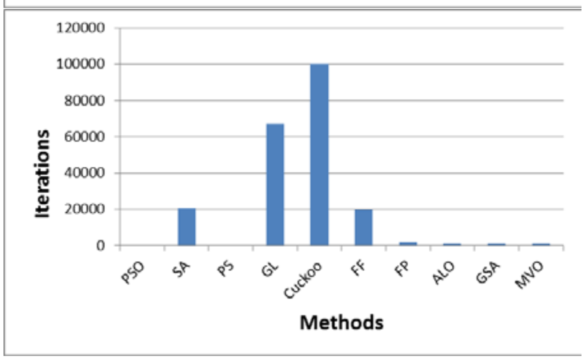

From the above graphs we know that the number of iteration and the elapsed time is minimum in PSO and PS But PS is high in other four parameters x_1, x_2, $x_{3,}$

4. Results and Discussion:

With the two extreme values of the parameters the optimization is carried out with different solvers. As they are stochastic type the results may vary from trial to trial. So the problem is made to run for 20 trials. (Elbeltagi.E., Tarek Hegazy.I., Grierson D., 2005) And an average of all trials is taken as a final value of the parameter by the solver. The solvers are compared with three different criteria.

4.1. Consistency

The weight is consistent in Pattern Search (3350.404)

4.2. Minimum run time:

For minimum run time of the problem we have PS (0.879797 seconds), PSO (0.817205 seconds).

4.3. Minimum Evaluation:

This Criterion will determine the effectiveness of the algorithm. From the table we see that the PS and PSO algorithm have minimum evaluation of 3 and 200 respectively.

4.4. The Simplicity of Algorithm:

Of all the algorithms, Pattern Search algorithm is the most simplest followed by Particle Swarm Optimization.

Thus it is seen that the PS solver satisfies all the criteria. Even though the pattern search satisfies all the above criteria, the weight becomes maximum whereas the weight in PSO is **3145.922**. Therefore the particle swarm optimization has the minimum weight with time 0.817205 seconds and 200 iteration so the appropriate algorithm for speed reducer design is suggested as Particle Swarm Optimization. It is apparent from the results that PSO algorithm is able to provide promising solutions with less objective function evaluations. This desirable characteristic of PSO algorithm would be more significant in one engineering problems which entail higher computational effort.

5. Conclusion:

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In the present study the PSO algorithm is proposed as a simple and efficient optimization technique for handling pressure vessel design problem. PSO algorithm is a population based technique which follows a stochastic iterative procedure to locate the optimum or a reasonably near- optimum solution for the pressure vessel design optimization. Performance evaluation of the PSO algorithm through pressure vessel design optimization reveals the efficiency of this technique in solving practical optimization problems. Although in the present study the PSO algorithm is utilized only for solving pressure vessel design optimization problem, it can be easily employed for solving other types of optimization problems as well.

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Tables for option set and Stopping criteria for the ten methods

methods	PSO	SA	PS	GL	CUCKOO	FF	FP	ALO	GSA	MVO
Option set	Max.Generation =200 Max.Time Limit=∞ Average change in fitness value=10^{-6} Function Tolerance: 10^{-6} Cognitive Attraction =0.5 Population Size=40 Social Attraction =1.25	Initial Temperature:100 Annealing Function: Fast Annealing Reannealing Interval :100 Time limit:∞ Max.Function Evaluation:3000*No.of variables. Max.Iteration: ∞ Function Tolerance:10^{-6} Objective Limit:10^{-6}	Poll Method:GPS positive basis 2N Initial Mesh Size:1 Expansion Function:2 Contraction Factor:0.5 Mesh Tolerence:10^{-6} Max.Function Evaluation:2000*No.of variables. Max.Iteration:100*No.of variables. Max.Time Limit:∞ Function Tolerance:10^{-6}	Max.Fun Evaluations= 10^{-5} Max.Iterations =20 Min.Iterations =2 Total Iterations=15 Functions Tolerance= 10^{-4}	Max.Fun. Evaluations =10^{-5} Max.Iterations=20 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun. Evaluations =10^{-5} Max.Iterations=20 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations =10^{-5} Max.Iterations=20 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations=10^{-5} Max.Iterations =20 Functions Tolerance=10^{-6} Max.Time Limit=∞	Max.Fun Evaluations =10^{-5} Max.Iterations=20 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations= 10^{-5} Max.Iterations=20 Functions Tolerance= 10^{-6} Max.Time Limit=∞
Stopping criteria	Max.Generation =200 Max.Time Limit=∞ Average change in fitness value=10^{-6} Function Tolerance: 10^{-6}	Max.Time reached The average change in value of the objective function is $< 10^{-6}$ max.iterations are reached if the number of functions evaluations reached. If the best objective function value is less than or equal to the value of objective limit.	Mesh Tolerance:10^{-6} Max.Iteration: 100*No.of variables. Evaluation:2000*No.of variables Max.Time Limit:∞ Function Tolerance:10^{-6}	Max.Fun Evaluations= 10^{-5} Max.Iterations =20 Min.Iterations =2 Total Iterations=15 Functions Tolerance= 10^{-4}	Max.Fun Evaluations =10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations =10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations =10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations= 10^{-6} Max.number of Iterations= 100000 Functions Tolerance=10^{-6} Max.Time Limit=∞	Max.Fun Evaluations =10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞	Max.Fun Evaluations= 10^{-6} Max.number of Iterations= 100000 Functions Tolerance= 10^{-6} Max.Time Limit=∞